

# Infrared Fixed Points for Ratios of Couplings in the Chiral Lagrangian

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## Abstract

In the framework of the low energy chiral Lagrangian the renormalization group equations for the couplings are investigated up to order  $p^6$  – as well for  $SU(2) \times SU(2)$  as for  $SU(3) \times SU(3)$  chiral symmetry. Infrared attractive fixed points for ratios of couplings are found. These fixed point solutions turn out to agree with the values determined from experiment in a surprisingly large number of cases.

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As is well known, an appropriate extension of QCD to low energies is an effective field theory which realizes the spontaneously broken (approximate) chiral symmetry nonlinearly in terms of the light Goldstone field degrees of freedom[1]. This symmetry information is encoded in the chiral Lagrangian. As in any effective field theory, the Lagrangian has infinitely many contributing operators; however, they may be arranged according to their importance for low energy observables in an expansion in powers of  $p/\Lambda$ . Here  $p$  denotes the low momentum scale of interest and  $\Lambda$  some momentum cut-off, above which the chiral Lagrangian ceases to be valid, with  $p/\Lambda \leq 1$ .

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The number of operators contributing to each order in this expansion is finite. Similarly, perturbation theory in the number of loops and the renormalization program can be carried out for effective field theories — even though in principle infinitely many counterterms are required. The choice of a mass-independent renormalization scheme ( $\overline{\text{MS}}$ ,  $\overline{\text{MS}}$ ) leads, however, to counterterms which may again be arranged in an expansion in powers of  $p/\Lambda$ . As a consequence, in any given order in  $p/\Lambda$  the number of counterterms needed to absorb the divergences is again finite[2].

Of interest are the coefficients of the operators in the chiral Lagrangian, or rather — after extraction of their dimension in form of powers of  $\Lambda$  — the dimensionless couplings. These couplings encode in principle the information about the QCD dynamics at higher scales. Unfortunately, in practice they are unknown. There have been efforts to estimate some of them by using different techniques (lattice calculations, large  $N_c$  limit, vector meson dominance, ...) [3]. The couplings in a given order of  $p/\Lambda$  have to be determined by experiment; one needs as many observables as couplings to be determined. Within the framework of perturbation theory and renormalization described above, the renormalization group equations for the couplings can be determined at any fixed order in  $p/\Lambda$ . They have all been calculated up to and including  $O((p/\Lambda)^4)$  — abbreviated by  $O(p^4)$  — and some of them<sup>1</sup> up to  $O(p^6)$  [4].

In this paper we investigate these renormalization group equations and search for infrared attractive fixed point solutions. This is a perfectly legitimate search, since the infrared limit probes small momenta, where the chiral Lagrangian is applicable. The analysis is performed as well for  $SU(2) \times SU(2)$  as for  $SU(3) \times SU(3)$  chiral symmetry. As it turns out, the renormalization group equations indeed exhibit infrared fixed points in *ratios* of couplings. These fixed points are non-trivial special solutions of the renormalization group equations which attract the renormalization group flow in its evolution from the scale  $\Lambda$  towards the infrared. It is very interesting to see, how they compare to the experimental values for the corresponding ratios of couplings (as far as these are available). This comparison is performed. Agreement is found for a surprisingly large number of ratios, thus confirming to a certain extent the fixed point solutions by data.

A first reference to fixed points of the linearly realized chiral Lagrangian was given in [6]. As the fixed point solutions correspond to renormalization group invariant relations between couplings, they may also be viewed as solutions of the parameter reduction program [7]. Earlier applications of the parameter reduction technique in the framework of effective field theories may be found in [8].

The effective chiral Lagrangian for chiral  $SU(3) \times SU(3)$  symmetry, expanded in

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<sup>1</sup>After completion of this work, we became aware of the new reference [5]. A more general investigation based on these impressive results is in preparation.

terms of increasing powers of  $p^2$ , may be written as follows

$$\begin{aligned}\mathcal{L}_\chi &= \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \dots \\ &= \mathcal{L}^{(2)} + \sum_i L_i \mathcal{O}_4^i + \sum_i \frac{K_i}{\Lambda^2} \mathcal{O}_6^i + \dots\end{aligned}\quad (1)$$

where  $\mathcal{O}_4$  and  $\mathcal{O}_6$  are dimension four and six operators, respectively, and where the dimensionful cut-off  $\Lambda$  is introduced, leading to dimensionless couplings  $L_i$  and  $K_i$ . The couplings to external right-handed and left-handed vector fields  $r_\mu, l_\mu$ , and scalar and pseudoscalar fields  $s, p$  are included. The lowest order,  $O(p^2)$ , Lagrangian  $\mathcal{L}^{(2)}$  may be written as

$$\mathcal{L}^{(2)} = \frac{F_0^2}{4} \langle D_\mu U D^\mu U^\dagger \rangle + \frac{F_0^2}{4} \langle \chi U^\dagger + U \chi^\dagger \rangle. \quad (2)$$

The operation  $\langle \cdot \rangle$  denotes the trace, the external fields enter through  $\chi = 2B_0(s + ip)$  and the covariant derivative  $D_\mu U = \partial_\mu U - ir_\mu U + iU l_\mu$ . The unitary matrix  $U$

$$U(\Phi) = \exp \left( i\sqrt{2}\Phi/F_0 \right) \quad (3)$$

is given, for  $SU(3) \times SU(3)$ , in terms of the Goldstone boson fields as follows

$$\Phi(x) \equiv \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -2\frac{\eta}{\sqrt{6}} \end{pmatrix}. \quad (4)$$

$\mathcal{L}^{(2)}$  involves two constants[9]  $F_0, B_0$ , which are undetermined by the symmetry. They are related to the pion decay constant and the quark condensate, respectively.

For the  $O(p^4)$  Lagrangian  $\mathcal{L}^{(4)}$  we follow the notation for the couplings  $L_i$  by Gasser and Leutwyler[9]. In the  $SU(3)$  case  $\mathcal{L}_4$  involves twelve terms:

$$\begin{aligned}\mathcal{L}^{(4)} &= \sum_{i=1}^{10} L_i \mathcal{O}_i + \sum_{i=1}^2 H_i \mathcal{O}'_i \\ &= L_1 \langle D_\mu U^\dagger D^\mu U \rangle^2 + L_2 \langle D_\mu U^\dagger D_\nu U \rangle \langle D^\mu U^\dagger D^\nu U \rangle + L_3 \langle D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U \rangle \\ &\quad + L_4 \langle D_\mu U^\dagger D^\mu U \rangle \langle U^\dagger \chi + \chi^\dagger U \rangle + L_5 \langle D_\mu U^\dagger D^\mu U (U^\dagger \chi + \chi^\dagger U) \rangle + L_6 \langle U^\dagger \chi + \chi^\dagger U \rangle^2 \\ &\quad + L_7 \langle U^\dagger \chi - \chi^\dagger U \rangle^2 + L_8 \langle U^\dagger \chi U^\dagger \chi + \chi^\dagger U \chi^\dagger U \rangle - iL_9 \langle F_R^{\mu\nu} D_\mu U D_\nu U^\dagger + F_L^{\mu\nu} D_\mu U^\dagger D_\nu U \rangle \\ &\quad + L_{10} \langle U^\dagger F_R^{\mu\nu} U F_{L\mu\nu} \rangle + H_1 \langle F_{R\mu\nu} F_R^{\mu\nu} + F_{L\mu\nu} F_L^{\mu\nu} \rangle + H_2 \langle \chi^\dagger \chi \rangle\end{aligned}\quad (5)$$

where the field strength tensors of the external gauge fields are  $F_{R\mu\nu} = \partial_\mu r_\nu - \partial_\nu r_\mu - i[r_\mu, r_\nu]$ , and similarly for  $l_\mu$ . The last two terms are not directly measurable, because they involve only the external sources. Correspondingly we shall not consider them in the following. For the  $SU(2)$  case three couplings are redundant.

The Lagrangian at order  $p^6$  involves about hundred terms[10], which we cannot write out in detail here.

Following the renormalization procedure for effective theories outlined in the introduction, we use dimensional regularization and a mass independent renormalization scheme. According to the renormalization program for the chiral Lagrangian[1], the relations between bare and renormalized couplings are:

$$L_i^b = \mu^{(d-4)} [L_i^r + \Gamma_i \lambda] \quad (7)$$

$$K_i^b = \mu^{2(d-4)} [K_i^r + (c_i + b_{ij} L_j^r) \lambda + a_i \lambda^2], \quad (8)$$

where  $\lambda = [(d-4)^{-1} - \zeta]/16\pi^2$ , and  $\zeta$  is a constant which depends on the renormalization scheme. The constants  $\Gamma_i$ ,  $c_i$ , and  $b_{ij}$  are calculable; they are scheme independent. It is now easy to obtain the renormalization group equations (RGE) for the renormalized couplings which have the following generic form

$$\mu \frac{dL_i^r}{d\mu} = -\frac{1}{16\pi^2} \Gamma_i \quad (9)$$

$$\mu \frac{dK_i^r}{d\mu} = -\frac{1}{16\pi^2} (c_i + b_{ij} L_j^r), \quad (10)$$

where summation over repeated indices is implied. The values for the constants  $\Gamma_i$  and for a selection of the constants  $c_i$  and  $b_{ij}$  for  $SU(2)$  and  $SU(3)$  symmetry, respectively, will be given further down. Henceforth we shall drop the index  $r$ , since we shall exclusively deal with renormalized quantities.

Let us next proceed with an analysis of the lowest order renormalization group equation (9) for infrared (IR) fixed points and their comparison to experiment. It is obvious that Eq. (9) has no IR fixed point. However, the RGE for the ratios  $L_i/L_j$  of couplings

$$\mu \frac{d}{d\mu} \left( \frac{L_i}{L_j} \right) = \frac{1}{16\pi^2} \frac{\Gamma_j}{L_j} \left( \frac{L_i}{L_j} - \frac{\Gamma_i}{\Gamma_j} \right) \quad (11)$$

has a fixed point at

$$\left. \frac{L_i}{L_j} \right|_{\text{f.p.}} = \frac{\Gamma_i}{\Gamma_j} \quad \text{for } \Gamma_i, \Gamma_j \neq 0. \quad (12)$$

The general solution of the RGE (9) for initial value  $L(\Lambda)$  at  $\mu = \Lambda$  is.

$$L_i(\mu) = L_i(\Lambda) - \frac{1}{16\pi^2} \Gamma_i \log \frac{\mu}{\Lambda} \quad (13)$$

One reads off that the approach of the ratio of  $L_i/L_j$  of general solutions  $L_i$ ,  $L_j$  to the IR fixed point is controlled by the *fast* approach of  $\log(\mu/\Lambda) \rightarrow -\infty$  in the IR limit  $\mu \rightarrow 0$ .

It is now interesting to compare the experimental values for the ratios of couplings with the predictions obtained for their fixed points (12).

The following tables summarize the experimental values of the couplings for  $SU(2)$  symmetry<sup>2</sup> [11, 9] at an energy scale of the order of the pion mass and for  $SU(3)$  symmetry [13] at an energy scale  $\mu = m_\rho$  (which is unfortunately rather large). These values are derived from meson decay constants, electromagnetic form factors and, for  $SU(3)$  symmetry, also from semileptonic kaon decays. Also the corresponding coefficients  $\Gamma_i$  of the beta functions are given.

$SU(2)$ :						
i	1	2	3	4	5	6
$l_i \cdot 10^3$	$-2.4 \pm 3.9$	$12.7 \pm 2.7$	$-4.6 \pm 3.8$	$27.2 \pm 5.7$	$-7.3 \pm 0.7$	$17.4 \pm 1.2$
$\Gamma_i$	1/3	2/3	-1/2	2	-1/6	-1/3

$SU(3)$ :					
i	1	2	3	4	5
$L_i \cdot 10^3$	$0.4 \pm 0.3$	$1.35 \pm 0.3$	$-3.5 \pm 1.1$	$-0.3 \pm 0.5$	$1.4 \pm 0.5$
$\Gamma_i$	3/32	3/16	0	1/8	3/8

i	6	7	8	9	10
$L_i \cdot 10^3$	$-0.2 \pm 0.3$	$-0.4 \pm 0.2$	$0.9 \pm 0.3$	$6.9 \pm 0.7$	$-5.5 \pm 0.7$
$\Gamma_i$	11/144	0	5/48	1/4	-1/4

In Figs. 1 and 2 the experimental ratios  $L_i/L_j$  are compared with their fixed point ratios  $\Gamma_i/\Gamma_j$  for values of  $i, j$  for which  $\Gamma_i$  and  $\Gamma_j$  are nonzero. In addition we omitted  $L_1$  for  $SU(2)$  and  $L_4$  and  $L_6$  for  $SU(3)$ , since their errors are too large to give meaningful comparisons. To guide the eye, we ordered the ratios from left to right with decreasing agreement of experimental values with the fixed point values. The overall agreement between experimental and predicted values is quite impressive. Even, where the agreement fails on a quantitative level (at the right end of the figures), the sign and the qualitative tendency are correct and the deviation is generically of the order of one standard deviation.

Next we proceed to the analysis of the  $O(p^6)$  RGE (10). We calculated the renormalization group equations for a selection of couplings  $K_i$ , based on results from Ref. [12] for  $SU(2)$  symmetry and from Ref. [14] for  $SU(3)$  symmetry. We use the notation of Refs. [10, 14].

For  $SU(2)$  symmetry the RGE, for a subset of the couplings, are:

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<sup>2</sup>The couplings  $l_i$  are defined as in Refs. [9, 11]. They are not identical with those given in Eq. (6).

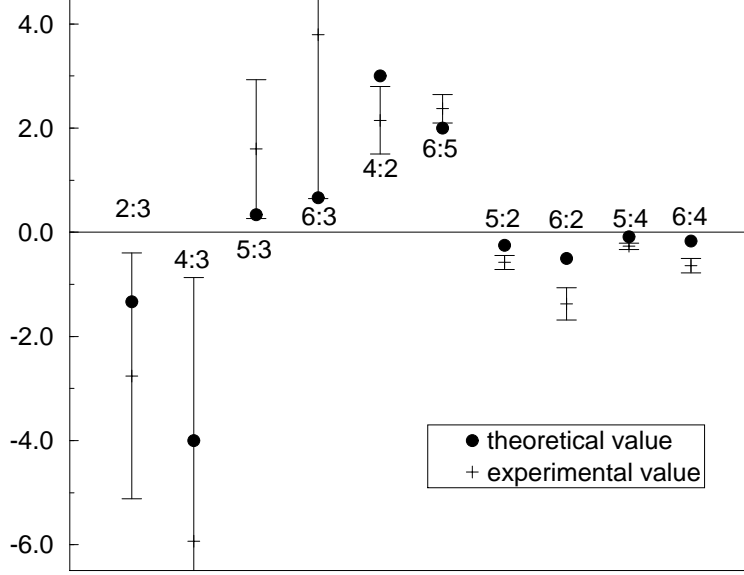


Figure 1: Ratios  $L_i/L_j$  of the experimental values of the couplings  $L_i$  in comparison with the corresponding IR fixed point ratios  $\Gamma_i/\Gamma_j$  for  $SU(2)$  symmetry. To guide the eye, the ratios are ordered from left to right according to decreasing agreement.

$$\begin{aligned}
\mu \frac{dK_1}{d\mu} &= \frac{1}{(16\pi^2)^2} \frac{193}{27} + \frac{1}{16\pi^2} \left( \frac{208}{3} L_1 + \frac{112}{3} L_2 + 24L_3 + 2L_4 \right) \\
\mu \frac{dK_2}{d\mu} &= -\frac{1}{(16\pi^2)^2} \frac{556}{27} - \frac{1}{16\pi^2} \left( 136L_1 + \frac{248}{3} L_2 + 28L_3 + 2L_4 \right) \\
\mu \frac{dK_3}{d\mu} &= \frac{1}{(16\pi^2)^2} \frac{755}{108} + \frac{1}{16\pi^2} \left( \frac{200}{3} L_1 + 44L_2 \right) \\
\mu \frac{dK_4}{d\mu} &= -\frac{1}{(16\pi^2)^2} \frac{1}{108} - \frac{1}{16\pi^2} \left( \frac{4}{3} L_1 + \frac{4}{3} L_2 \right) \\
\mu \frac{dK_5}{d\mu} &= -\frac{1}{(16\pi^2)^2} \frac{29}{432} - \frac{1}{16\pi^2} \left( \frac{21}{2} L_1 + \frac{107}{12} L_2 \right) \\
\mu \frac{dK_6}{d\mu} &= -\frac{1}{(16\pi^2)^2} \frac{79}{432} - \frac{1}{16\pi^2} \left( \frac{5}{6} L_1 + \frac{25}{12} L_2 \right)
\end{aligned} \tag{14}$$

And for  $SU(3)$  symmetry we consider the RGE:

$$\begin{aligned}
\mu \frac{dK}{d\mu} &= -\frac{1}{(16\pi^2)^2} \frac{43}{96} - \frac{1}{16\pi^2} \left( \frac{104}{9} L_1 + \frac{26}{9} L_2 + \frac{61}{18} L_3 - \frac{34}{9} L_4 + L_5 - 4L_6 - 2L_8 \right) \\
\mu \frac{dA}{d\mu} &= \frac{1}{(16\pi^2)^2} \frac{175}{288} + \frac{1}{16\pi^2} \left( \frac{28}{3} L_1 + \frac{34}{3} L_2 + \frac{25}{3} L_3 - \frac{26}{3} L_4 + \frac{8}{3} L_5 + 12L_6 - 12L_8 \right)
\end{aligned}$$

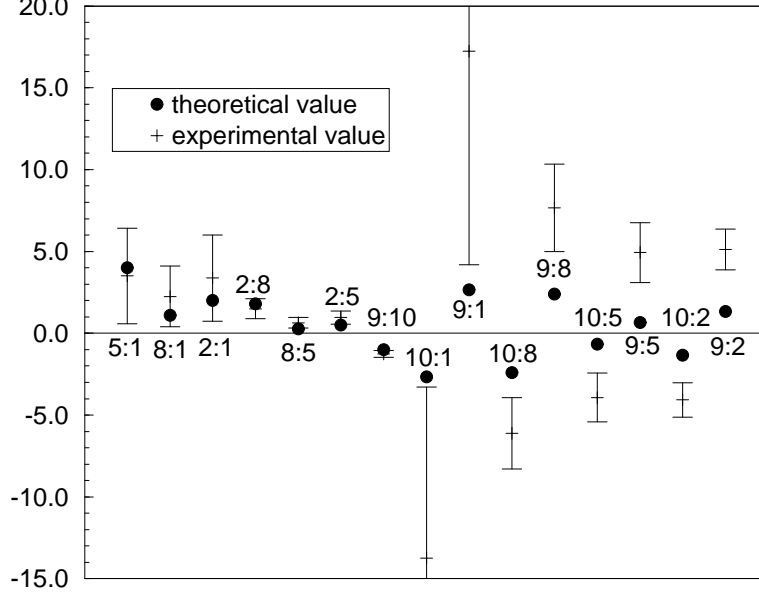


Figure 2: As in Fig. 1, but for  $SU(3)$ .

$$\begin{aligned}
\mu \frac{dB}{d\mu} &= -\frac{1}{(16\pi^2)^2} \frac{19}{48} + \frac{1}{16\pi^2} \left( \frac{32}{9} L_1 + \frac{8}{9} L_2 + \frac{8}{9} L_3 - \frac{106}{9} L_4 + \frac{22}{9} L_5 + 20 L_6 \right) \\
\mu \frac{dC}{d\mu} &= -\frac{1}{(16\pi^2)^2} \frac{691}{2592} - \frac{1}{16\pi^2} \left( \frac{28}{9} L_1 + \frac{34}{9} L_2 + \frac{59}{18} L_3 - \frac{26}{9} L_4 + 3 L_5 + 4 L_6 - 6 L_8 \right) \\
\mu \frac{dD}{d\mu} &= -\frac{1}{(16\pi^2)^2} \frac{20}{27} + \frac{1}{16\pi^2} \left( \frac{20}{3} L_4 + \frac{23}{3} L_5 - \frac{40}{3} L_6 - 40 L_7 - \frac{86}{3} L_8 \right) \\
\mu \frac{dE}{d\mu} &= \frac{1}{(16\pi^2)^2} \frac{5}{162} + \frac{1}{16\pi^2} \left( \frac{44}{9} L_4 + \frac{1}{27} L_5 - \frac{88}{9} L_6 - \frac{136}{9} L_7 - \frac{134}{27} L_8 \right) \\
\mu \frac{dF}{d\mu} &= \frac{1}{(16\pi^2)^2} \frac{167}{144} - \frac{1}{16\pi^2} \left( 8 L_6 - 64 L_7 - \frac{62}{3} L_8 \right) \\
\mu \frac{dG}{d\mu} &= -\frac{1}{(16\pi^2)^2} \frac{371}{648} - \frac{1}{16\pi^2} \left( 2 L_5 - \frac{16}{3} L_6 + 72 L_7 + 24 L_8 \right) \\
\mu \frac{dH}{d\mu} &= \frac{1}{(16\pi^2)^2} \frac{9}{16} + \frac{1}{16\pi^2} \left( L_5 + \frac{152}{3} L_7 + \frac{62}{3} L_8 \right)
\end{aligned} \tag{15}$$

where we defined, as in Ref. [14], the following combinations of couplings:

$$K = B_{19} + B_{21} ; \quad A = 2B_{14} - B_{17} - 3B_{19} + 3B_{21} ; \quad B = B_{16} + B_{18} - 2B_{19} + 2B_{21} \tag{16}$$

$$C = B_{15} - B_{20} + B_{19} - B_{21} ; \quad D = B_3 + B_4 + B_5 + 3B_7 + \frac{1}{2}B_1 + \frac{1}{3}B_2 \tag{17}$$

$$E = B_6 - \frac{1}{3}B_4 - \frac{1}{3}B_5 - B_7 - \frac{1}{6}B_1 - \frac{1}{9}B_2 ; \quad F = B_{14} - \frac{3}{2}B_4 - \frac{3}{2}B_5 - \frac{9}{2}B_7 \tag{18}$$

$$G = B_{15} + B_1 + \frac{2}{3}B_2 + B_4 + 2B_5 + 3B_7; \quad H = B_{16} - \frac{1}{2}B_1 - \frac{1}{3}B_2 - 2B_4 - B_5 - 3B_7. \quad (19)$$

The general solution of the generic RGE (10) for the  $O(p^6)$  couplings  $K_i$  in terms of the general solution for the  $O(p^4)$  couplings  $L_i$  is

$$K_i(\mu) = K_i(\Lambda) - \frac{1}{16\pi^2} (c_i + b_{ij}L_j(\Lambda)) \log \frac{\mu}{\Lambda} + \frac{1}{2} \frac{1}{(16\pi^2)^2} b_{ij}\Gamma_j \left( \log \frac{\mu}{\Lambda} \right)^2. \quad (20)$$

There is first of all a renormalization group invariant relation between the  $O(p^6)$  couplings  $K_i$  and the  $O(p^4)$  couplings  $L_i$ , *an IR attractive fixed line*. For all ratios  $L_i/L_j$  given by their respective IR fixed points  $L_i/L_j = \Gamma_i/\Gamma_j$  the relation is

$$K_i = (c_i + \frac{1}{2}b_{ij}L_j) \frac{L_m}{\Gamma_m}. \quad (21)$$

This predicts the  $K_i$  in terms of the  $L_i$ . For the index  $m$  any suitable value can be chosen, since all the ratios  $L_m/\Gamma_m$  are equal in the fixed point. This is a very interesting and strong prediction.

Obviously, the ratios  $K_i/K_j$  have IR fixed points for all ratios  $L_i/L_j$  sitting in their respective fixed point solutions (12)

$$\left. \frac{K_i}{K_j} \right|_{\text{f.p.}} = \frac{c_i + (1/2)b_{ik}\Gamma_k}{c_j + (1/2)b_{jk}\Gamma_k} \quad \text{for} \quad c_i + (1/2)b_{ik}\Gamma_k, \quad c_j + (1/2)b_{jk}\Gamma_k \neq 0. \quad (22)$$

All the IR fixed point and fixed line predictions can also be obtained within the parameter reduction program.

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